(7 pages) **Reg. No. : ....**

Code No. : 40331 E **Sub. Code : JMMA 11/ JMMC 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

First Semester

Mathematics/ Mathematics with CA – Main

CALCULUS

(For those who joined in July 2016 only)

Time : Three hours Maximum : 75 marks

PART A — $(10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The locus of the centre of curvature for a curve is ————.

   (a) Involute (b) Evolute

   (c) Complete (d) Regular

2. If $u = x^3 + y^3 + 3x^2y + 3xy^2$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(a) u (b) $2u$

(c) $3u$ (d) $-u$

3. Pedal equation of the circle with radius a is

(a) $\rho = \frac{r^2}{a}$ (b) $\rho = \frac{r^2}{a^2}$

(c) $\rho^2 = \frac{r}{a}$ (d) None

4. General equation of the asymptotes of a hyperbola is

(a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (b) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 0$ (d) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

5. $\int_0^{\frac{\pi}{2}} \sin^5 x\, dx =$

(a) $\frac{8}{15}$ (b) $\frac{4}{15}$

(c) $\frac{8\pi}{15}$ (d) $\frac{4\pi}{15}$

6. $\int_0^1 \int_0^2 xy \, dx \, dy =$

(a) 0

(b) $\frac{1}{6}$

(c) 1

(d) $\frac{3}{2}$

7. If $x = u(1+v), y = v(1+u)$ then $\frac{\partial(x,y)}{\partial(u,v)} =$ ________.

(a) $1+u+v$

(b) $u+v$

(c) $1-u+v$

(d) $1-u-v$

8. Change the order of integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dx \, dy$

(a) $\int_0^1 \int_x^y \frac{e^{-y}}{y} dx \, dy$

(b) $\int_0^\infty \int_0^x \frac{e^{-y}}{y} dx \, dy$

(c) $\int_0^1 \int_0^y \frac{e^{-y}}{y} dx \, dy$

(d) None of these

9. $\left[\Gamma\left(\frac{1}{2}\right)\right]^2 =$

(a) π (b) $\frac{\pi}{2}$

(c) $\sqrt{\pi}$ (d) $\frac{\sqrt{\pi}}{2}$

10. $\beta(2, 3) =$ ________.

(a) $\frac{1}{2}$ (b) $\frac{5\pi}{32}$

(c) $\frac{1}{12}$ (d) $\frac{3}{8}$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Derive the Cartesian formula for the radius of curvature.

Or

(b) Find the radius of curvature of the Cardioid $r = a(1 - \cos\theta)$.

12. (a) Prove that the $p-r$ equation of the cardioid $r = a(1-\cos\theta)$ is $\rho^2 = \frac{r^3}{2a}$.

Or

(b) Find the asymptote of $x^3 + y^3 = 3axy$.

13. (a) Show that $x^4 - 2x^2y - xy^2 - 2x^2 - 2xy + y^2 - x + 2y + 1 = 0$ has a single cusp of the second kind at (0, –1).

Or

(b) Trace the curve $y = \frac{x}{(2-x)^2}$.

14. (a) Evaluate $\iint (x^2 + y^2)\,dx\,dy$ over the region for which x, y are each ≥ 0 and $x + y \leq 1$.

Or

(b) If $x + y = u$, $y = uv$, change the variables to u, v in the integral $\iint [xy(1-x-y)]^{\frac{1}{2}}\,dx\,dy$ taken over the area of the triangle with sides $x = 0, y = 0$, $x + y = 1$ and evaluate it.

15. (a) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

Or

(b) Evaluate $\int_0^1 x^m \left(\log \frac{1}{x}\right)^n dx$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Prove that the radius of curvature at a point $(a\cos^3\theta, a\sin^3\theta)$ on the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is $3a \sin\theta \cos\theta$.

Or

(b) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

17. (a) Find the $p-r$ equation of the curve $x^2 + y^2 = ax$ and deduce its radius of curvature.

Or

(b) Find the rectilinear asymtotes of $2x^4 - 5x^2y^2 + 3y^4 + 4y^3 - 6y^3 + x^2 + y^2 - 2xy + 1 = 0$.

18. (a) Examine for double points of the curve $x^4 - 2ay^3 - 3a^2y^2 - 2a^2x^2 + a^4 = 0$.

Or

(b) Trace the curve $x^3 + y^3 = 3axy$.

19. (a) Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\, dx\, dy$ and evaluate it.

Or

(b) Evaluate $\iiint xyz\, dx\, dy\, dz$ over the +ve octant of the sphere $x^2 + y^2 + z^2 = a^2$ by transformation into spherical coordinates.

20. (a) Prove that $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$.

Or

(b) Prove that $\int_0^\infty x^2 e^{-x^8} dx \times \int_0^\infty x^2 e^{-x^4} dx = \frac{\pi}{16\sqrt{2}}$.
