(6 pages) **Reg. No. :** ....

Code No. : 40568 E **Sub. Code : SMMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

First Semester

Mathematics — Core

CALCULUS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. For any curve, the curvature is ————.

(a) $\frac{ds}{d\chi}$ (b) $\frac{d\chi}{ds}$

(c) $\frac{dy}{dx}$ (d) $\frac{ds}{dx}$

2. The radius of curvature of a circle of diameter d is

(a) d (b) $\frac{2}{d}$

(c) $\frac{d}{2}$ (d) $2d$

3. The $p-r$ equation of a parabola is ————.

(a) $p^2 = ar$ (b) $p = ar$

(c) $p = a^2 r$ (d) $p^2 = ar^2$

4. The number of asymptotes of a general curve of n^{th} degree is

(a) $n+1$ (b) $n-1$

(c) $2n$ (d) n

5. If a double point to the curve $f(x, y) = 0$ is a conjugate point then ————.

(a) $f_{xy}^2 < f_{xx} \cdot f_{yy}$ (b) $f_{xy}^2 > f_{xx} \cdot f_{yy}$

(c) $f_{xy}^2 = f_{xx} \cdot f_{yy}$ (d) None

6. The curve $xy = c^2$ is symmetrical about ————.

(a) x–axis (b) y–axis

(c) $x = y$ (d) both the axes

7. The value of $\iint dy\,dx$ over the region $x \geq 0$; $y \geq 0$; $x + y \leq 1$ is

(a) $\frac{1}{3}$ (b) $-\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

8. Value of $\int_0^a \int_0^b \int_0^c dx\,dy\,dz$ is

(a) $a+b+c$ (b) abc

(c) $\frac{a+b+c}{2}$ (d) $a-b-c$

9. $\int_1^\infty \frac{dx}{x^2} =$ ____________.

(a) 1 (b) 0

(c) ∞ (d) 2

10. $\int_0^1 x^2(1-x)^3\,dx =$ ____________.

(a) $\frac{1}{5}$ (b) $\frac{1}{10}$

(c) 1 (d) $\frac{1}{60}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that the radius of curvature of the curve $y = c\cosh\left(x/c\right)$ at any point is $\frac{y^2}{c}$.

Or

(b) Find the formula for finding the radius of curvature of a curve expressed in polar form.

12. (a) Find the $(p-r)$ equation of the curve $r = a\sin\theta$.

Or

(b) Find all the asymptotes of the curves $x^3 - xy^2 + 6y^2 = 0$.

13. (a) Find the position and nature of the double points of the curve $x^2(x-y) + y^2 = 0$.

Or

(b) Trace the curve $y^2 = \frac{x^2(a+x)}{b-x}$.

14. (a) Evaluate $\iint xy\,dx\,dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.

Or

(b) By changing into polar co-ordinates, evaluate $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2+y^2)\,dx\,dy$.

[P.T.O.]

15. (a) Evaluate $\int_{-1}^{1} \frac{dx}{x}$.

Or

(b) Evaluate $\int_{0}^{\infty} e^{-x^2} dx$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that the radius of curvature of $r^n = a^n \cos n\theta$ is $\frac{a^n r^{-n+1}}{n+1}$.

Or

(b) Show that in the parabola $y^2 = 4ax$ at the point 't', $\rho = -2a(1+t^2)^{3/2}$; $X = 2a + 3at^2$ and $Y = -2at^3$.

17. (a) Prove that the evolute of the cycloid $x = a(\theta - \sin\theta)$; $y = a(1-\cos\theta)$ is another cycloid.

Or

(b) Find the asymptoms of $x^3 + 2x^2y - 4xy^2 - 8y^3 - 4x + 8y = 1$.

18. (a) Prove that the curve $x^4 = y^2(x+y)$ has a double cusp of the first species at the origin.

Or

(b) Trace the curve $y = (x-1)(x-2)(x-3)$.

19. (a) Evaluate $\iiint xyz\, dx\, dy\, dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

Or

(b) Change the order of integration in $\int_0^a \int_y^a \frac{x\, dy\, dx}{x^2 + y^2}$ and evaluate it.

20. (a) Define $\Gamma(n)$ and prove that $\Gamma(n+1) = n!$ where n is a positive integer. Also test the convergence of $\Gamma(n)$.

Or

(b) State and prove relation between Beta and Gamma functions.