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Reg. No. :..

Code No. : 20710 E **Sub. Code : AMMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

First Semester

Mathematics — Core

CALCULUS AND CLASSICAL ALGEBRA

(For those who joined in July 2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The curvature of the curve $ax + by + c = 0$ is ____________.

   (a) b (b) a

   (c) 0 (d) none of the above

2. The radius of curvature of $y = e^x$ at $(0, 1)$ is ____________.

   (a) 1 (b) 2

   (c) $2\sqrt{2}$ (d) none of the above

3. The value of $\int_1^b \int_1^a \frac{dxdy}{xy}$ = ___________.

(a) $\log\left(\frac{a}{b}\right)$ (b) $\log(ab)$

(c) $\log a \log b$ (d) none of the above

4. The Jacobian of $u = x + y$ and $v = x - y$ is ___________.

(a) 2 (b) 1

(c) –2 (d) none of the above

5. $\int_0^1 x^2(1-x)dx$ = ___________.

(a) 2 (b) $\frac{1}{12}$

(c) $\frac{1}{3}$ (d) none of the above

6. $\int_0^{\pi}\int_0^{\pi/2}\int_0^1 r^2 \sin\theta dr d\theta d\phi$ = ___________.

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) none of the above

7. The least degree of the equation with rational coefficients one of whose roots $\sqrt{2}+\sqrt{3}$ is ___________

(a) 3 (b) 2

(c) 4 (d) none of the above

8. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$ then $\sum \frac{1}{\alpha}$ = ___________.

(a) $-\frac{q}{r}$ (b) $\frac{q}{r}$

(c) $\frac{p}{r}$ (d) none of the above

9. The roots of the equation $x^n + 1 = 0$ (n is even) are ___________.

(a) all imaginary (b) $(n-1)$ imaginary

(c) $(n-2)$ imaginary (d) none of the above

10. One of the roots of the equation $2x^3 + 3x^2 - 3x - 2 = 0$ is –2, the other roots are ___________.

(a) –2, –1 (b) $-\frac{1}{2}, 1$

(c) $-\frac{1}{2}, -1$ (d) none of the above

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the p-r equation (pedal equation) of the curve $r^2 = a^2 \sin 2\theta$.

Or

(b) Find the coordinates of the center of curvature of the curve $x^3 + y^3 = 3axy$ at $\left(\frac{a}{2}, \frac{a}{2}\right)$.

12. (a) Find the area of the region common to $y^2 = 4ax$ and $x^2 = 4ay$.

Or

(b) If $u = 2xy$, $v = x^2 - y^2$, $x = r\cos\theta$, $y = r\sin\theta$, evaluate $\frac{\partial(u, v)}{\partial(r, \theta)}$ without actual substitution.

13. (a) Prove that $\Gamma\left(\frac{n+1}{2}\right) = \frac{(2n)!\sqrt{\pi}}{4^n n!}$ where $n = 0, 1, 2, \ldots$

Or

(b) Prove that

$$\int_0^{\pi/2} \sin^p \theta \cos^q \theta d\theta = \frac{1}{2}\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right).$$

14. (a) Show that the sum of the 6th powers of the roots of $x^7 - x^4 + 1 = 0$ is 3.

Or

(b) If α, β, γ are the roots of the equation $x^3 + ax^2 + bx + c = 0$, form the equation whose roots are $\alpha\beta$, $\alpha\gamma$ and $\beta\gamma$.

15. (a) Transform the equation $x^4 - 4x^3 - 18x^2 - 3x + 2 = 0$ into an equation with the third term absent.

Or

(b) Remove the fractional coefficient from the equation $x^3 + \frac{1}{4}x^2 - \frac{1}{16}x + \frac{1}{72} = 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find the coordinates of the center of curvature of $y = x \log x$ at the point where $\frac{dy}{dx} = 0$.

Or

(b) Find the evolute of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

17. (a) By changing the order of integration, evaluate the integral $\int_0^1 \int_y^{2-y} xy\,dx\,dy$.

Or

(b) By changing into polar coordinates, show that $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\,dy = \frac{\pi}{4}$. Hence evaluate $\int_0^\infty e^{-t^2} dt$.

18. (a) Evaluate $\int_0^1 x^m (1-x^n)^p\,dx$ in terms of gamma functions and hence find $\int_0^1 \frac{dx}{\sqrt{1-x^n}}$.

Or

(b) Using gamma functions evaluate $\int\int xy(1-x-y)^{1/2} dx\,dy$ over the area enclosed by the lines $x=0$, $y=0$ and $x+y=1$ in the positive quadrant.

19. (a) Solve $6x^2 - 11x^2 + 6x - 1 = 0$ where roots are in harmonic progression.

Or

(b) If $a + b + c + d = 0$, show that

$$\frac{a^5 + b^5 + c^5 + d^5}{5} = \frac{a^2 + b^2 + c^2 + d^2}{2} \cdot \frac{a^3 + b^3 + c^3 + d^3}{3}$$

20. (a) Show that the equation $x^4 - 3x^3 + 4x^2 - 2x + 1 = 0$ can be transformed into a reciprocal equation by diminishing the roots by unity. Hence solve the given equation.

Or

(b) Solve the equation

$$6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0.$$