(8 pages) Reg. No. : .....

Code No. : 41151 E **Sub. Code : JMMA 12/ JMMC 12/SMMA 12**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

First Semester

Mathematics / Mathematics with CA — Main

CLASSICAL ALGEBRA

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. If the equation $2x^3 - 3x^2 + 2x - 3 = 0$ has one root 'i' then, its real root is _____

(a) $\frac{2}{3}$

(b) $-\frac{2}{3}$

(c) $\frac{3}{2}$

(d) 1

2. The smallest degree of an equation with rational coefficients two of whose roots are $2+3i$ and $2-3i$ roots is _____

(a) 2 (b) 4

(c) 6 (d) 3

3. The sum of the roots of the equation $x^4 - ax^3 + bx^2 - cx + d = 0$ is _____

(a) $-\frac{b}{a}$ (b) $\frac{b}{a}$

(c) a (d) $-a$

4. A reciprocal equation $a_0 x^n + a_1 x^{n-1} + ... + a_n = 0$ is said to be of second type is _____

(a) $a_{n-r} = a_{r-1}$

(b) $a_{n-r} = a_{r+1}$

(c) $a_{n-r} = a_r$

(d) $a_{n-r} = -a_r$

5. To remove the second term of $x^4 - 12x^3 + 48x^2 - 72x + 35 = 0$ the roots are to be diminished by ———————

(a) 1 (b) 2

(c) 3 (d) -1

6. If the roots of $x^3 - 8x^2 + 19x - 12 = 0$ are 1, 3, 4 then the roots of $x^3 - 16x^2 + 76x - 96 = 0$ are ———————

(a) 1, 3, 4 (b) $-1, -3, -4$

(c) 2, 6, 8 (d) 1, 9, 16

7. The negative roots of $f(x) = 0$ are ———————

(a) positive roots of $f(-x) = 0$

(b) positive roots of $f(-x) = -1$

(c) positive roots of $f(+x) = 0$

(d) negative roots of $f(-x) = 0$

8. If all the roots of $f(x) = 0$ are real then all the roots of $f'(x) = 0$ are ———————

(a) imaginary

(b) real

(c) real or imaginary

(d) positive

9. Cardon's method deals with solving a ———————

(a) quadratic equation

(b) cubic equation

(c) bi quadratic equation

(d) quintic equation

10. For the cubic equation $x^3 - 6x - 4 = 0$, the value of the discriminate is ———————

(a) -16 (b) 14

(c) 52 (d) 48

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) If one root of the equation $2x^3 - 11x^2 + 38x - 39 = 0$ is $2 - 3i$. Solve the equation.

Or

(b) Solve the equation $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots are in arithmetic progression.

12. (a) If $\alpha + \beta + \gamma = 6$, $\alpha^2 + \beta^2 + \gamma^2 = 14$ and $\alpha^3 + \beta^3 + \gamma^3 = 36$ prove that, $\alpha^4 + \beta^4 + \gamma^4 = 98$.

Or

(b) Show that $4\left(x^2 - x + 1\right)^3 = 27x^2\,(x-1)^2$ is a standard reciprocal equation.

13. (a) Increase the roots of the equation $3x^4 + 7x^3 - 15x^2 + x - 2 = 0$ by 7.

Or

(b) Discuss the reality of the roots $x^4 + 4x^3 - 2x^2 - 12x + a = 0$ for all values of a.

14. (a) Find the multiple roots of $x^5 - x^4 + 2x^3 - 2x^2 + x - 1 = 0$ and hence solve.

Or

(b) Obtain by Newton's method the root of the equation $x^3 - 3x + 1 = 0$ which lies between 1 and 2.

15. (a) Solve $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$ using Ferrari's method.

Or

(b) Solve $2x^3 + 3x^2 + 3x + 1 = 0$ by Cardan's method.

SECTION C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) One root of the equation $2x^6 - 3x^5 + 5x^4 + 6x^3 - 27x + 81 = 0$ is $\sqrt{2} + i$. Find the remaining roots.

Or

(b) Show that the roots of the equation $px^3 + qx^2 + rx + s = 0$ are in G.P. iff $r^3 p = q^3 s$.

17. (a) State and prove Newton's theorem.

Or

(b) Solve: $6x^5 + 11x^4 - 33x^2 + 11x + 6 = 0$.

18. (a) State and prove Rolle's theorem.

Or

(b) Find the nature of the roots of $x^4 + 4x^3 - 20x^2 + 10 = 0$.

19. (a) Find the strum's functions for the polynomial $x^4 - 2x^3 - 3x^2 + 10x - 4$.

Or

(b) Find the positive root of the equation $x^3 - 2x^2 - 3x - 4 = 0$ correct to three places of decimals.

20. (a) Solve $x^3 - 3x^2 - 10x + 24 = 0$ using Cardan's method.

Or

(b) Solve $4x^4 + 8x^3 + 12x^2 + 4x + 5 = 0$ using Ferrari's method.
