(8 pages) **Reg. No. :** ....

Code No. : 40345 E **Sub. Code : JAMA 21/ SAMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Second/Fourth Semester

Mathematics — Allied

VECTOR CALCULUS AND FOURIER SERIES

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $\overline{A} = u^2\vec{i} + u\vec{j} + 2u\vec{k}$ and $\overline{B} = \vec{j} - u\vec{k}$ then $\frac{d}{du}\left(\overline{A} \cdot \overline{B}\right)$ is

   (a) $2u - 1$ (b) $2u + 1$

   (c) $1 - 4u$ (d) $1 + 4u$

2. If $\bar{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then $\nabla \times \bar{r}$ is

(a) 0
(b) 1
(c) 2
(d) 3.

3. $\int_0^1 \int_0^2 xy^2 \, dy \, dx =$

(a) $\frac{1}{3}$
(b) $\frac{2}{3}$
(c) 1
(d) $\frac{4}{3}$

4. $\int_0^{\pi} \int_0^1 r^4 \sin\theta \, dr \, d\theta$

(a) $\frac{1}{5}$
(b) $\frac{2}{5}$
(c) $\frac{3}{5}$
(d) .1

5. If $\bar{f} = x^2\vec{i} - xy\vec{j}$ and C is the straight line joining the points (0, 0) and (1, 1) then $\int_C \bar{f} \cdot d\bar{r} =$

(a) 1
(b) 0
(c) –1
(d) 2

6. The value of $\iint dx\, dy$ over the region bounded by $x = 0,\ x = 2,\ y = 0;\ y = 2$ is

(a) 2 (b) 4

(c) 0 (d) 3

7. If R is any closed region of the xy-plane bounded by a simple closed curve C then $\int_C y\, dx + x\, dy$ is

(a) 1 (b) 0

(c) π (d) 2π

8. Green's theorem connects

(a) line integral and double integral

(b) line integral and surface integral

(c) double integral and surface integral

(d) surface integral and volume integral.

9. An example of an even function is

(a) x (b) $|x|$

(c) $x + x^3$ (d) $x + x^2$

10. The Fourier coefficient a_0 for the function $f(x) = x \sin x$ in $(0, 2\pi)$ is

(a) 0 (b) 1

(c) 2 (d) –2.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Find ϕ if

$$\nabla\phi = \left(6xy + z^3\right)\vec{i} + \left(3x^2 - z\right)\vec{j} + \left(3xz^2 - y\right)\vec{k}.$$

Or

(b) Prove that $curl\left(\overline{r} \times \overline{a}\right) = -2\overline{a}$, where $\overline{a}$ is a constant vector.

12. (a) Evaluate $\int_0^{\pi/2}\int_0^{\pi/2} \sin(x + 2y)\, dx\, dy$.

Or

(b) Evaluate $\int_0^a\int_0^b\int_0^c (x + y + z)\, dx\, dy\, dz$.

13. (a) Evaluate $\int_C \overline{f} \cdot d\overline{r}$, where

$\overline{f} = (2y + 3)\vec{i} + xz\vec{j} + (yz - x)\vec{k}$ and C is the straight line joining the points (0, 0, 0) and (2, 1, 1).

Or

(b) Evaluate $\iint_S \bar{f} \cdot \hat{n} dS$ where

$\bar{f} = (x + y^2)\bar{i} - 2x\bar{j} + 2yz\bar{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

14. (a) By using Stoke's theorem, prove that $\int_C \bar{r} \cdot d\bar{r} = 0$ where $\bar{r} = x\vec{i} + y\vec{j} + z\vec{k}$.

Or

(b) If $\bar{f} = x^2\bar{i} + y^2\bar{j} + z^2\bar{k}$ and V is the volume enclosed by the cube $0 \le x, y, z \le 1$ then evaluate $\iiint_V \nabla \cdot \bar{f} \, dV$.

15. (a) Find the Fourier series for the function

$$f(x) = \begin{cases} -x & -\pi \le x < 0 \\ x & 0 \le x \le \pi. \end{cases}$$

Or

(b) Find the Fourier sine series for the function $f(x) = k$ in the interval $0 < x < \pi$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Prove that $div\left(r^n\bar{r}\right) = (n+3)\,r^n$. Deduce that $r^n\bar{r}$ is solenoidal iff $n = -3$.

Or

(b) Prove that

$$curl\left(\bar{f}\times\bar{g}\right) = (\bar{g}\cdot\nabla)\bar{f} - (\bar{f}\cdot\nabla)\bar{g} + \bar{f}\,div\,\bar{g} - \bar{g}\,div\,\bar{f}$$

17. (a) Find the area of the circle $x^2 + y^2 = r^2$ by using double integral.

Or

(b) Evaluate $\iiint\limits_D \frac{dx\,dy\,dz}{(x+y+z+1)^3}$ where D is the region bounded by the planes $x=0$; $y=0$; $z=0$ and $x+y+z=1$.

18. (a) Evaluate $\iint\limits_S \left(\nabla\times\bar{f}\right)\cdot\hat{n}\,dS$ where $\bar{f} = y^2\vec{i} + y\vec{j} - xz\vec{k}$ and S is the upper half of the sphere $x^2+y^2+z^2=a^2$ and $z\geq 0$.

Or

(b) Find $\int_C \bar{f} \cdot d\bar{r}$ where $\bar{f} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$ and C is

(i) the straight line from (0, 0, 0) to (2, 1, 3).

(ii) the curve $x = 2t^2$; $y = t$; $z = 4t^2 - 1$ from $t = 0$ to $t = 1$.

(iii) the curve $x^2 = 4y$; $3x^2 = 8z$ from $x = 0$ to $x = 2$.

19. (a) Verify Green's theorem for

$$\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy,$$

where C is the boundary of the region R enclosed by $x = 0$; $y = 0$; $x + y = 1$.

Or

(b) Verify Gauss divergence theorem for $\bar{f} = y\vec{i} + x\vec{j} + z^2\vec{k}$ for the cylindrical region S given by $x^2 + y^2 = a^2$; $z = 0$ and $z = 4$.

20. (a) Find the Fourier series for the function $f(x) = x^2$ in the interval $-\pi \le x \le \pi$ and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.

Or

(b) (i) Prove that the Fourier cosine series for the function $f(x)=x$ in the interval $0 \leq x \leq \pi$ is

$$x=\frac{\pi}{2}-\frac{4}{\pi}\left[\frac{\cos x}{1^2}+\frac{\cos 3x}{3^2}+\frac{\cos 5x}{5^2}+\cdots\right].$$

Hence deduce that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{8}.$$

(ii) Prove that the Fourier sine series for the function $f(x)=x$ in the interval $0 \leq x \leq \pi$ is

$$x=2\left[\frac{\sin x}{1}-\frac{\sin 2x}{2}+\frac{\sin 3x}{3}-\cdots\right].$$

Hence deduce that $1-\frac{1}{3}+\frac{1}{5}-\cdots=\frac{\pi}{4}$.
