(8 pages) **Reg. No. : ....**

Code No. : 40014 E **Sub. Code : GAMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2019.

Second/Fourth Semester

Mathematics – Allied

VECTOR CALCULUS

(For those who joined in July 2012 – 2015)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ then $\nabla \cdot \bar{r} =$ ————————.

   (a) 1

   (b) 0

   (c) 3

   (d) $x^2 + y^2 + z^2$

2. The vector function $f = x^2\bar{i} + y^2\bar{j} + z^2\bar{k}$ is ________.

(a) solenoidal

(b) irrotational

(c) harmonic

(d) neither solenoidal nor irrotational

3. $\int e^{ax+b}dx =$ ________.

(a) $\frac{1}{b}e^{ax+b}$

(b) $\frac{1}{a}e^{ax+b}$

(c) $\frac{1}{a}e^{bx+a}$

(d) $\frac{1}{b}e^{bx+a}$

4. $\int \cot\theta \, d\theta =$ ________.

(a) $\log\sin\theta$

(b) $\log\tan\theta$

(c) $\tan\theta$

(d) $\sin\theta$

5. The value of $\iint dx\,dy$ over the region bounded by $x = 0, x = 2, y = 0, y = 2$, is ________.

(a) 2

(b) 4

(c) 0

(d) 3

6. The value of $\int_0^a \int_0^a \int_0^a dz\,dy\,dx$ is ________.

(a) a^3 (b) a^2

(c) a (d) 1

7. If C is the straight line joining $(0, 0, 0)$ and $(1, 1, 1)$ then $\int_C \bar{r} \cdot dr$ is ________.

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{3}{2}$ (d) 2

8. If $\bar{f} = (x^2 + y^2)\bar{i} + (x^2 - y^2)\bar{j}$ then the value of $\int_C r \cdot dr$ where C is the part of the curve $y = x^2$ joining the points $(0, 0)$ and $(1, 1)$ is ________.

(a) 0 (b) $\frac{9}{10}$

(c) $\frac{1}{2}$ (d) 2

9. If V is the volume enclosed by the closed surface S then the value of $\iint_S \overline{r} \cdot \overline{n}\, dS$ is ————.

(a) $3V^2$ (b) $3V$

(c) $6V$ (d) 0

10. Gauss's divergence theorem connects

(a) line integral and double integral

(b) line integral and surface integral

(c) double integral and surface integral

(d) surface integral and volume integral

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $\overline{r} = \overline{a} \cos \omega t + \overline{b} \sin \omega t$ where $\overline{a}, \overline{b}$ are constant vectors and ω is a constant prove that $\overline{r} \times \frac{d\overline{r}}{dt} = \omega(\overline{a} \times \overline{b})$ and $\frac{d^2\overline{r}}{dt^2} + \omega^2 \overline{r} = 0$.

Or

(b) Show that $div\left(\frac{\overline{r}}{r}\right) = \frac{2}{r}$.

12. (a) Evaluate $\int \frac{x^2}{(a+bx)^3} dx$.

Or

(b) Evaluate $\int \frac{dx}{(1+e^x)(1+e^{-x})}$.

13. (a) Evaluate $I = \int_0^{\pi} \int_0^{a\cos\theta} \bar{r} \sin\theta \, dr \, d\theta$.

Or

(b) Evaluate $\int_0^2 \int_1^3 \int_1^2 xy^2 z \, dz \, dy \, dx$.

14. (a) If $\bar{f} = x^2 \bar{i} - xy \bar{j}$ and C is the straight line joining the points $(0, 0)$ and $(1, 1)$ find $\int_C f \cdot dr$.

Or

(b) Evaluate $\iint_S (x^2 + y^2) dS$ where S is the surface of the cone $z^2 = 3(x^2 + y^2)$ bounded by $z = 0$ and $z = 3$.

15. (a) Verify Gauss divergence theorem for the vector function $\bar{f} = (x^3 - yz)\bar{i} - 2x^2y\bar{j} + 2\bar{k}$ over the cube bounded by $x = 0, y = 0, z = 0$, $x = a, y = a$ and $z = a$.

Or

(b) Verify Stokes theorem for the vector function $\bar{f} = y^2\bar{i} + y\bar{j} - xz\bar{k}$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ and $z \geq 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find the equation of the

(i) tangent plane and

(ii) normal line to the surface $xyz = 4$ at the point $(1, 2, 2)$.

Or

(b) Prove that $curl(curl f) = grad\ div\ f - \nabla^2 f$.

17. (a) Evaluate $\int \frac{x}{\sqrt{x^2+x+1}} dx$.

Or

(b) Evaluate $\int \frac{dx}{(3+x)\sqrt{x}}$.

18. (a) Evaluate $I = \iint_D xy\,dy\,dx$ where D is the region bounded by the curve $x = y^2, x = 2 - y, y = 0$ and $y = 1$.

Or

(b) Find by triple integral the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

19. (a) If $\bar{f} = (2y+3)\,\bar{i} + xzj + (yz - x)\bar{k}$ evaluate $\int_C \bar{f} \cdot d\bar{r}$ along the following paths C

(i) $x = 2t^2; y = t; z = t^3$ from $t = 0$ to $t = 1$

(ii) The polygonal path P consisting of the three lines segments AB, BC, CD where
$A = (0, 0, 0), B = (0, 0, 1), C = (0, 1, 1)$ and $D = (2, 1, 1)$.

(iii) The straight line joining $(0, 0, 0)$ and $(2, 1, 1)$.

Or

(b) Evaluate $\iint_S (\nabla \times \bar{f}) \cdot \bar{n}\, dS$ where $\bar{f} = y^2 \bar{i} + y\, \bar{j} - xz\bar{k}$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$ and $z \geq 0$.

20. (a) Verify Gauss divergence theorem for the function $\bar{f} = a(x + y)\bar{i} + a(y - x)\bar{j} + z^2\bar{k}$ over the hemisphere bounded by the $x \circ y$ plane and the upper half of the sphere $x^2 + y^2 + z^2 = a^2$.

Or

(b) Using Green's theorem evaluate $\int_C (xy - x^2)dx + x^2 y dy$ along the closed curve C formed by $y = 0, x = 1$ and $y = x$.
